

Experimentally accessible witnesses of many-body localisation

M. Goihl,¹ M. Friesdorf,¹ A. H. Werner,¹ W. Brown,¹ and J. Eisert¹

¹*Dahlem Center for Complex Quantum Systems, Freie Universität Berlin, D-14195 Berlin, Germany*

(Dated: January 19, 2016)

The phenomenon of many-body localised (MBL) systems has attracted significant interest in recent years, for its intriguing implications from a perspective of both condensed-matter and statistical physics: they are insulators even at non-zero temperature and fail to thermalise, violating expectations from quantum statistical mechanics. What is more, recent seminal experimental developments with ultra-cold atoms in optical lattices constituting analog quantum simulators have pushed many-body localised systems into the realm of physical systems that can be measured with high accuracy. In this work, we introduce experimentally accessible witnesses that directly probe distinct features of MBL, distinguishing it from its Anderson counterpart. We insist on building our toolbox from techniques available in the laboratory, including on-site addressing, super-lattices, and time-of-flight measurements, identifying witnesses based on fluctuations, density-density correlators, densities, and entanglement. We build upon the theory of out of equilibrium quantum systems, in conjunction with tensor network and exact simulations, showing the effectiveness of the tools for realistic models.

Many-body localisation provides a puzzling and exciting paradigm within quantum many-body physics and is for good reasons attracting significant attention in recent years. Influential theoretical work¹ building upon the seminal insights by Anderson on disordered models² suggested that localisation would survive the presence of interactions. Such many-body localised models, as they were dubbed, would be insulators even at non-zero temperature and exhibit no particle transport. Maybe more strikingly from the perspective of statistical physics, these many-body localised models would fail to thermalise following out of equilibrium dynamics^{3–5}, challenging common expectations how systems “form their own heat bath” and hence tend to be locally well described by the familiar canonical Gibbs ensemble^{6–8}. Following these fundamental observations, a “gold rush” of theoretical work followed, identifying a plethora of phenomenology of such many-body localised models. They would exhibit a distinct and peculiar logarithmic scaling of entanglement in time^{9,10}, the total correlations of time averages have a distinct scaling¹¹, many Hamiltonian eigenstates fulfil area laws¹² for the entanglement entropy^{13,14} and hence violate what is called the eigenstate thermalisation hypothesis¹⁵. The precise connection and interrelation between these various aspects of many-body localisation is just beginning to be understood^{14,16–19}, giving rise to a vivid discussion in theoretical physics.

These theoretical studies have recently been complemented by seminal experimental activity, allowing to probe models that are expected to be many-body localised in the laboratory under remarkably controlled conditions^{20,21}. This work goes much beyond earlier demonstrations of Anderson localisation in a number of models²², in that now actual interactions are expected to be relevant. Such ultra-cold atomic systems indeed provide a pivotal arena to probe the physics that is at stake here²³. What is still missing, however, is a direct detection of the rich phenomenology of many-body localisation in the laboratory. Rather than seeing localisation and taking the presence of interactions for granted, it seems highly desirable to make use of these novel exciting possibilities to directly see the above features, distinctly separating the observations from those expected from non-interacting Anderson insulators. Such a mindset is that of “witnessing” a property,

somewhat inspired by how properties like entanglement are witnessed^{24–26} in quantum information.

In this work, we aim at capturing precisely those aspects of the rich phenomenology of many-body localisation that are directly accessible with present experimental tools. We would like to provide a “dictionary” of possible tools, as a list or a classification of features that can be probed making use of only in situ site resolved measurements, including the measurement of density-density correlation and time of flight measurements, in conjunction with a variation of densities. In this way, we aim at identifying a comprehensive list of features that “could be held responsible” for MBL, based on data alone. While all we explicitly state is directly related to cold atoms in optical lattices, a similar approach is expected to be feasible in continuous cold bosonic atoms on atom chips^{27,28}, where correlation functions of all orders can readily be directly measured. We leave this as an exciting perspective.

Probing disordered optical lattice systems

The setting we focus on is that of interacting (spin-less) fermions placed into a one-dimensional optical lattice, a setting that prominently allows to probe the physics under consideration^{20,23}. Such systems are well described by

$$H = \sum_j \left(f_j^\dagger f_{j+1} + \text{h.c.} \right) + \sum_j w_j n_j + U \sum_j n_j n_{j+1}, \quad (1)$$

where f_j denotes a fermionic annihilation operator on site j and $n_j = f_j^\dagger f_j$ is the local particle number operator. The disorder in the model is carried by the local potential-strength w_j , which is drawn independently at each lattice site j according to a suitable probability distribution. Experimentally, the disorder can either be realised by superposing the lattice with an incommensurate laser or by speckle patterns²⁰. From Eq. (1) one obtains the disordered Heisenberg chain²⁹ by setting $U = 2$ and scaling the disorder by a factor two. To keep the discussion conceptually clear, just as Ref.²⁹ we make use of a uniform distribution on the interval $[-I, I]$, where we refer to I as the disorder strength. Thus, for $U = 2$ the ergodic

to MBL phase transition is approximately at $I \approx 7^{29}$. The phase diagramme is best known for $U = 0$ corresponding to the non-interacting Anderson insulator and for $U = 2$, the MBL phase. To add a flavour of usual phase transitions order parameters such as total correlations¹¹, fluctuations of local observables³⁰ or the structure of the eigenstates³¹ have been suggested. While these quantities impressively signal the transition, it is not a priori clear whether they can be implemented in an actual experiment. Recent numerical studies³² show that pump-probe type setups and novel instances of spin noise spectroscopy³³ as well as utilising MBL systems as a bath³⁴ are indeed suited to distinguish the above phases, albeit experimental realisations of this endeavour appear to need substantial changes and innovations in realistic setups. Another possibility for the phase distinction, which has prominently been carried out experimentally²¹, is given by observing the behaviour of quasi two-dimensional systems in comparison to their one dimensional counterparts. While this impressively demonstrates the capabilities of optical lattices as platforms for quantum simulations, it does not test the properties of MBL in one dimension as such. We set out to find comparably strong and direct signatures of one dimensional MBL, which however rely on simple established measurement operations. Hence, we start by summarising the measurements, which we conceive to be feasible in an optical lattice experiment.

Measurements considered feasible

We now turn to specifying what measurements we consider feasible in optical lattices with state of the art techniques. For this, we focus on the following two types of measurements:

In-situ: An in-situ measurement detects the occupation of individual lattice sites. This technique only allows to resolve the parity of the particle number on each site, which for fermions constitutes no limitation, however. Using the fact that single-shot measurements are performed, also higher moments like density-density correlators can be extracted from this kind of measurements. Both ramifications will be used.

Time-of-flight: The time-of-flight (ToF) measurement extracts position-averaged momentum information of the form

$$\langle n(q, t_{\text{ToF}}) \rangle = |\hat{w}_0(q)|^2 \sum_{j,k} e^{iq(r_j - r_k) - i \frac{c(r_j^2 + r_k^2)}{t_{\text{ToF}}}} \langle f_j^\dagger f_k \rangle,$$

where $\{r_j\}$ are the positions of the lattice sites, \hat{w}_0 reflects the Wannier functions in momentum space, and $c > 0$ is a constant derived from the mass of the particles and the lattice constant.

The main goal of this work is to identify key quantities that indicate that the system indeed is many-body localised based on measurement information extracted using these two techniques. Here, we want to show both that the system is localised and that it is interacting. Thus, we also want to convincingly detect the difference between an MBL system and a non-interacting Anderson insulator. In order to approach this task, we will look at the time evolution of an initial state that is particularly easy to prepare experimentally relying on optical

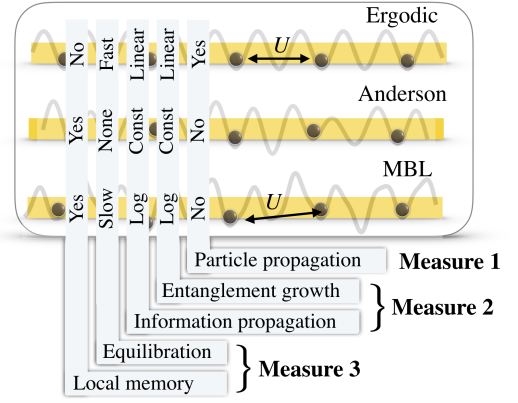


FIG. 1. An overview over the dynamical behaviour of MBL systems versus their ergodic and thermalising and Anderson localised counterparts. **Measure 1** detects particle propagation and phase correlations and can be implemented using ToF imaging. **Measure 2** and **Measure 3** utilise in-situ imaging to observe density-density correlations and equilibration behaviour.

super-lattices^{20,35}, namely an alternating pattern of the form

$$|\psi(t=0)\rangle = |0, 1, 0, 1, \dots, 0, 1\rangle. \quad (2)$$

This initial product state will, during time evolution, build up entanglement and become correlated^{9,10}. Naturally, this is far from being the only choice for an initial state and alterations in this pattern and correspondingly locally changing particle and hole densities would surely be insightful, specifically since a modulation of the density already points towards interactions in the MBL phase being significant. In this work, we put emphasis on measurements although preparation procedures as the above mentioned density variations are an interesting problem in its own right. However, as we will demonstrate, the above defined initial state already captures the colourful phenomenology of MBL in all of its salient aspects.

Phenomenology of many-body localisation

A fundamental characteristic of MBL is the presence of local constants of motion³. They are approximately local operators $\tilde{\sigma}_j^z$ whose support is centered on lattice site j , but which nevertheless commute with the Hamiltonian, i.e., $[H, \tilde{\sigma}_j] = 0$. These operators are mainly supported on a region with diameter ξ , corresponding to the localisation length scale of the system. In fact, in the MBL regime, the dynamics can be captured by a phenomenological model in terms of a set of mutually commuting quasi-local constants of motion,

$$H_{l-\text{Bit}}^{(2)} = \sum_i \mu_i \tilde{\sigma}_i^z + \sum_{j < i} J_{i,j} \tilde{\sigma}_i^z \tilde{\sigma}_j^z. \quad (3)$$

Such a Hamiltonian constitutes a second order approximation of what is known as the l -bit model^{37,38} to exemplify the dynamics. Here $\tilde{\sigma}_j^z$ again denotes a quasi-local integral of motion centred on site i , μ_i is a random onsite potential and the

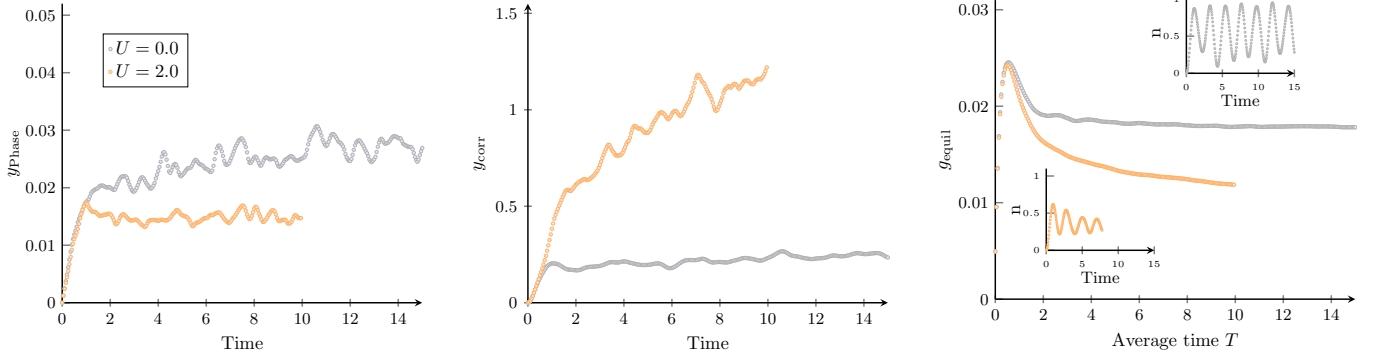


FIG. 2. Plotted are the results of a TEBD simulation³⁶ of the dynamical evolution of the initial state ψ from Eq. (2) under the Hamiltonian in Eq. (1) for the case of an Anderson insulators with $U = 0$ and MBL with $U = 2$. The disorder strength is $I = 8$. The three plots are averaged over 100 disorder realisations. **Left:** Shown is the time evolution of y_{Phase} defined in **Measure 1** demonstrating that the phase correlation behaviour saturates both for MBL and Anderson localisation. **Center:** The plot shows the dynamical evolution of y_{Corr} defined in **Measure 2**. Information propagation is fully suppressed in an Anderson insulator, resulting in a saturation of this quantity. In contrast, correlations continue to spread in the MBL system beyond all bounds, giving rise to a remarkably strong signal feasible to be detected in experiments. **Right:** Shown are the averaged fluctuations g_{Eq} defined in **Measure 3** as a function of the time T over which the average is performed. The insets show the time evolution of the particle density at the position $L/2$, which enters the calculation of g_{Eq} for one disorder realisation which is identical for the MBL and Anderson localised model. As the insets also show, the local fluctuations continue indefinitely for the Anderson insulator, corresponding to a saturation of g_{Eq} , while the MBL system equilibrates and g_{Eq} continues to decrease accordingly.

coupling strength $J_{i,j}$ between constants of motion is assumed to decay suitably fast in their distance $d(i, j)$. In particular, it is expected that the dynamics generated by the Hamiltonian defined in Eq. (1) in the MBL regime corresponding to $U = 2$ and $I > 7$ can be well captured by the l -bit model.

This phenomenological model gives rise to a separation of time scales in the evolution into two regimes. Initially, there is a fast regime, where the evolution takes place mainly inside the support of each local constant of motion $\tilde{\sigma}_i^z$. Hence, for this time scale, transport is unconstrained and particles and energies can move freely over the localisation length. Correspondingly, information can spread ballistically. Beyond the localisation length, the dynamics is dominated by the coupling of the constants of motion, given by the second term in Eq. (3)³⁷. The intuition is that this evolution does not facilitate particle or energy propagation, leading to a complete breakdown of thermal and electric conductivity. Nevertheless, the couplings between distant constants of motion allow for the creation of correlations over arbitrary length scales given sufficient time. This dephasing mechanism in turn makes it possible to send information and yields an explanatory mechanism for the observed slow growth of entanglement^{9,10,16}, measured as the von Neumann entropy of the half chain of an infinite system $S(t) = \Theta(\log(t))$ (in Landau notation).

Mathematically, these two dynamical regimes are best distinguished by the effect of a local unitary excitation on distant measurements. More precisely, given a local measurement O_A supported in a spatial region A and a unitary V_B corresponding to a local excitation in a region B , we wish to bound the change in expectation value of $O_A(t)$ induced by the unitary excitation. This can be cast into a Lieb-Robinson

bound^{14,39} of the form

$$\left| \langle V_B O_A(t) V_B^\dagger \rangle - \langle O_A(t) \rangle \right| \leq C(A) \begin{cases} e^{-\mu(d(A,B)-v|t|)} & \text{I,} \\ t e^{-\mu d(A,B)} & \text{II,} \end{cases} \quad (4)$$

where $C(A)$ a constant depending on the support of A . For the connection between different zero velocity Lieb-Robinson bounds and the necessity of a linear t -dependence in **II**, see Ref.¹⁴. Here **I** corresponds to the ballistic regime and **II** captures the slower dephasing. In the context of optical lattices, local excitations seem difficult to implement. Hence, in the following, we focus on the observation of indirect effects on the dynamical evolution in MBL systems.

Feasible witnesses

In the following, we demonstrate that local memory of initial conditions, slow spreading of correlations and equilibration of local densities provide clear measures to distinguish MBL systems from both the non-interacting Anderson insulators and also from ergodic systems, i.e., those where local measurements, after a short relaxation time, can be captured by thermal ensembles. In order to carry out our analysis, we will complement the intuitive guideline provided by the phenomenological l -bit model by a numerical tensor network TEBD simulation⁴⁰ (for details see Appendix A). The chosen parameters for the simulation are a disorder strength of $I = 8$ and interaction strengths of $U = 2$ or $U = 0$ for the MBL and Anderson case, respectively. We begin by considering the influence of the suppression of particle propagation.

1. Absence of particle transport

A defining feature of localised systems is that independent of the interaction strength, particles and energies do not spread over the entire system, but remain confined to local regions. They merely redistribute inside the localisation length, which can be extracted from the constants of motion. Therefore, even for long times, the particle density profile of an MBL system will not move to its thermal form, but rather retain some memory of its initial configuration. This gives rise to the following particle localisation measure.

Measure 1 (Particle propagation and phase correlations)

We define the following measure $y_{\text{Phase}}(t)$, which probes particle propagation for a system of length L

$$f_{\text{Phase}}(k, t) := \left| \langle f_{L/2}^\dagger(t) f_{L/2+k}(t) \rangle \right|, \quad (5)$$

$$y_{\text{Phase}}(t) = \sum_k f_{\text{Phase}}(k, t) k^2. \quad (6)$$

On an intuitive level, this measure directly probes the spread of particles, including weights based on the distance to the initial position $L/2$ such that distant contributions are amplified.

Numerically, we find that $y_{\text{Phase}}(t)$ initially shows a steep linear increase, indicative of the ergodic dynamics governed by the onsite term of Eq. (3) (Fig. 2). In the second regime it fluctuates without visible growth, indicating a break-down of particle transport on length scales beyond the localisation length. Thus, the length scale of the phase correlations established in the system can be bounded independent of time $y_{\text{Phase}}(t) = O(1)$. For ergodic systems, where particles and energies spread ballistically, the measure would grow in an unconstrained fashion over time. Based on this insight, we deduce that time-of-flight images, while clearly distinguishing between localised and ergodic phase, are not useful for the distinction between interacting and non-interacting localised systems.

Again more formally, this measure can be understood by considering the time evolution of the correlation matrix given by the matrix elements

$$\gamma_{j,k}(t) := \langle f_j^\dagger(t) f_k(t) \rangle, \quad (7)$$

where $\langle f_j^\dagger f_k \rangle = \text{Tr}(f_j^\dagger(t) f_k(t) \rho)$. For the non-interacting case of an Anderson insulator, this evolution is unitary $\gamma(t) = U(t) \gamma(0) U^\dagger(t)$, where $f_j^\dagger(t) = \sum_l U_{j,l}(t) f_l^\dagger$ is the evolution of the fermionic mode operators. For an Anderson insulator, dynamical localisation precisely corresponds to locality of the unitary evolution⁴¹, meaning that the matrix elements of U are expected to decay exponentially $|U_{j,k}(t)| \leq C e^{-d(j,k)}$ for some constant C with high probability⁴².

In the case of interacting Hamiltonians that conserve the particle number, this time evolution can be captured in form of a quantum channel

$$\gamma(t) = \sum_{l=1}^{L^2} K_l(\rho_0, t) \gamma(0) K_l^\dagger(\rho_0, t), \quad (8)$$

where the Kraus operators $K_l(\rho_0, t)$ depend on the full initial state. As particle propagation in an MBL system is expected to also be localised, it is assumed that the individual Kraus operators obey $|K_{j,k}(\rho_0, t)| \leq C_K e^{-d(j,k)}$. Starting from an initial product state of the form in Eq. (2), we obtain

$$\begin{aligned} \gamma_{z_1, z_2}(t) &= \langle f_{z_1}^\dagger(t) f_{z_2}(t) \rangle \\ &= \sum_{j,l} \langle z_1 | K(\rho_0, t) | j \rangle \gamma(0)_{j,l} \langle l | K(\rho_0, t) | z_2 \rangle \\ &= \sum_{j \text{ even}} C_K^2 e^{-d(j,0) - d(z_1+j, z_2)}. \end{aligned} \quad (9)$$

This again results in a suppression with the distance between z_1 and z_2 , causing a saturation of the phase correlation measure $f_{\text{Phase}}(k, t)$ independent of time.

2. Slow spreading of information

While particles and energies remain confined in interacting localised systems, correlations are expected to show an unbounded increase over time^{9,10}, although slower than in the ergodic counterpart. In stark contrast, Anderson localised many-body systems will not build up any correlations that go beyond the localisation length. In order to probe the spreading of correlations in the system, we focus on a quantity easily accessible in the context of optical lattices, using in-situ images for different evolution times. As it turns out, this kind of simple density-density correlator is already sufficient to separate Anderson localisation from MBL systems.

Measure 2 (Logarithmic information propagation) In order to examine the spatial spreading of density-density correlations, we define the quantity $y_{\text{Corr}}(t)$,

$$f_{\text{Corr}}(k, t) := |\langle n_{L/2} n_{L/2+k} \rangle - \langle n_{L/2} \rangle \langle n_{L/2+k} \rangle|, \quad (10)$$

$$y_{\text{Corr}}(t) := \sum_k f_{\text{Corr}}(k, t) k^2. \quad (11)$$

y_{Corr} is a direct indicator for the length scale over which density-density correlations are established without having to resort to assuming an explicit form, such as a decay in terms of an exponential function.

Comparable to the dynamics of the phase correlations, we numerically find a steep initial increase followed by a saturation for the non-interacting case (Fig. 2). The MBL system however, continues to build up density-density correlations for the times simulated. There is a transition in propagation speed, which we ascribe to the two dynamical regimes discussed before. Hence we conclude, that density-density correlations can be used to discriminate MBL from its non-interacting counterpart.

An intuitive explanation for the spread of density-density correlations despite spatial localisation of particles is that after exploring the localisation length, the particles feel the presence of neighbouring particles. Mediated by this interaction, the local movement of particles, governed by the respective

constant of motion, becomes correlated, even over large distances. In contrast, in the Anderson insulator where constants of motion are completely decoupled, this communication cannot take place.

We can connect this intuitive explanation to the more rigorous setting of Lieb-Robinson-bounds. In the Anderson insulator in one-dimension, there provably exists a zero-velocity Lieb-Robinson-bound, where the correlator on the left hand side of Eq. (4) is bounded by a time independent factor $e^{-\mu d(A,B)}$. This means that the detectability of an excitation created in region A decreases exponentially with the distance to B . On the contrary, in the MBL regime we expect a logarithmic Lieb-Robinson cone of the form of Eq. (4) II. Hence, an unbounded growth of correlations between distant regions is in principle possible, given sufficient time. Furthermore, we have shown that this built-up of correlations does also happen on observable time scales as can be seen from the evolution of density-density correlations captured by **Measure 2**.

3. Dephasing and equilibration

It is also instructive to study the differences between the Anderson and MBL-regime with respect to their equilibration properties. Due to the interactions present, we expect equilibration of fluctuations to take place in MBL systems, whereas in Anderson insulators the effective subspaces explored by single particles remain small for all times and hence fluctuations remain large. This in turn implies that fluctuations of local expectation values die out in the interacting model, but persist in an Anderson insulator. This qualitative difference has already been identified as a signifier of interactions in a disordered system²⁰. Here, we build upon this idea and propose to consider the average change rate of local expectation values in order to detect the decreasing fluctuations in the MBL phase.

Measure 3 (Density evolution: Equilibration of fluctuations)

We consider the expectation value $f_{\text{Eq}}(t) = \langle n_{L/2} \rangle(t)$ of a local density operator in the middle of the system. As a measure of local equilibration, we introduce the averaged rate of change of this density as a function of time $T > 0$

$$g_{\text{Eq}}(T) := \frac{1}{T} \int_0^T dt |f'_{\text{Eq}}(t)|. \quad (12)$$

As laid out in Fig. 2 again, this function over time indeed shows a remarkably smooth behaviour that allows for the clear distinction between an Anderson localised system and its MBL counterpart in that after a mutual increase the Anderson system saturates at a constant value, whereas in the MBL phase, g_{Eq} shrinks successively.

If we again resort to the Lieb-Robinson bound picture, we find that in the Anderson case a local excitation is confined

to a distinct spatial region given by the zero-velocity Lieb-Robinson-bound introduced in the previous chapter. This implies that the effective subspace explored is constant and specifically, the excitation cannot build up long distance correlations and fluctuations remain large. This can also be seen from the results of **Measure 2**. If we now, however, turn to the interacting model, a local excitation will slowly explore larger and larger parts of the Hilbert space, leading to a slow, but persistent decrease of the fluctuations.

Conclusions and outlook

In this work, we proposed an operational procedure for distinguishing MBL phases building upon realistic measurements, which can be performed in the realm of optical lattices with present technology. Utilising a phenomenological model and the concept of Lieb-Robinson-bounds, we explained the effects numerically investigated employing tensor network methods. The equilibration of local observables allows for the distinction of Anderson and MBL localised models. Density-density correlations allow for the same information bit extraction, while furthermore reproduce the expected phenomenology. Further investigating this quantity might yield information about the localisation length via the duration of the first evolution regime.

Phase correlations, which are directly connected to ToF imaging, cannot detect interactions in a localised system due to their correspondence to particle transport. There is yet other information the ToF reveals: One can also lower bound the spatial entanglement of bosons in optical lattices⁴³, building upon the ideas of constructing quantitative entanglement witnesses^{24–26}, a notion of multi-partite entanglement $M(t)$ detecting a deviation from a best separable approximation, as $M(t) \geq \max(0, \langle n \rangle - \langle n(q) \rangle / |\hat{w}_0(q)|^2)$ for all q . This quantity detects a reasonable notion of multi-particle entanglement, which is yet different from the bi-partite entanglement discussed above. Since this measure is only onsite local, we would expect that it cannot distinguish the long-range correlations of an interacting disordered model from the dynamics inside the constant of motion. This further motivates the quest to engineer appropriate entanglement witnesses both accessible in optical lattice architectures as well as probing key features of MBL, a quest that is in turn expected to contribute to our understanding of MBL as such.

Acknowledgements

We would like to thank the EU (AQUS, SIQS, RAQUEL), the ERC (TAQ), the BMBF (Q.com), and the DFG (SPP 1798) for support, and U. Schneider, C. Gross, A. Scardicchio, and R. Vasseur for discussions.

¹ D. M. Basko, I. L. Aleiner, and B. L. Altshuler, Ann. Phys. **321**, 1126 (2006).

² P. W. Anderson, Phys. Rev. **109**, 1492 (1958).

- ³ R. Nandkishore and D. A. Huse, *Ann. Rev. Cond. Matt. Phys.* **6**, 15 (2015).
- ⁴ A. Pal and D. A. Huse, *Phys. Rev. B* **82**, 174411 (2010).
- ⁵ V. Oganesyan and D. A. Huse, *Phys. Rev. B* **75**, 155111 (2007).
- ⁶ A. Polkovnikov, K. Sengupta, A. Silva, and M. Vengalattore, *Rev. Mod. Phys.* **83**, 863 (2011).
- ⁷ J. Eisert, M. Friesdorf, and C. Gogolin, *Nature Phys.* **11**, 124.
- ⁸ C. Gogolin and J. Eisert, “Equilibration, thermalisation, and the emergence of statistical mechanics in closed quantum systems,” (2015), arXiv:1503.07538.
- ⁹ M. Znidaric, T. Prosen, and P. Prelovsek, *Phys. Rev. B* **77**, 064426 (2008).
- ¹⁰ J. H. Bardarson, F. Pollmann, and J. E. Moore, *Phys. Rev. Lett.* **109**, 017202 (2012).
- ¹¹ J. Goold, S. R. Clark, C. Gogolin, J. Eisert, A. Scardicchio, and A. Silva, *Phys. Rev. B* **92**, 180202(R) (2015).
- ¹² J. Eisert, M. Cramer, and M. B. Plenio, *Rev. Mod. Phys.* **82**, 277 (2010).
- ¹³ B. Bauer and C. Nayak, *J. Stat. Mech.* **P09005** (2013).
- ¹⁴ M. Friesdorf, A. H. Werner, W. Brown, V. B. Scholz, and J. Eisert, *Phys. Rev. Lett.* **114**, 170505 (2015).
- ¹⁵ M. Srednicki, *Phys. Rev. E* **50**, 888 (1994).
- ¹⁶ I. H. Kim, A. Chandran, and D. A. Abanin, *Phys. Rev. B* **91**, 085425 (2015).
- ¹⁷ A. Chandran, J. Carrasquilla, I. H. Kim, D. A. Abanin, and G. Vidal, *Phys. Rev. B* **92**, 024201 (2015).
- ¹⁸ M. Friesdorf, A. H. Werner, M. Goihl, J. Eisert, and W. Brown, *New J. Phys.* **17**, 113054 (2015).
- ¹⁹ M. Serbyn, Z. Papić, and D. A. Abanin, *Phys. Rev. B* **90**, 174302 (2014).
- ²⁰ M. Schreiber, S. S. Hodgman, P. Bordia, H. P. Lüschen, M. H. Fischer, R. Vosk, E. Altman, U. Schneider, and I. Bloch, *Science* **349**, 842 (2015).
- ²¹ P. Bordia, H. P. Lüschen, S. S. Hodgman, M. Schreiber, I. Bloch, and U. Schneider, “Coupling identical 1d many-body localised systems,” (2015), arXiv:1509.00478.
- ²² D. S. Wiersma, P. Bartolini, A. Lagendijk, and R. Righini, *Nature* **390**, 671 (1997).
- ²³ I. Bloch, J. Dalibard, and S. Nascimbene, *Nature Phys.* **8**, 267 (2012).
- ²⁴ J. Eisert, F. G. S. L. Brandao, and K. M. Audenaert, *New J. Phys.* **9** (2007).
- ²⁵ K. M. R. Audenaert and M. B. Plenio, *New J. Phys.* **8**, 266 (2006).
- ²⁶ O. Guehne, M. Reimpell, and R. F. Werner, *Phys. Rev. Lett.* **98**, 110502 (2007).
- ²⁷ M. Gring, M. Kuhnert, T. Langen, T. Kitagawa, B. Rauer, M. Schreitl, I. Mazets, D. A. Smith, E. Demler, and J. Schmiedmayer, *Science* **337**, 1318 (2012).
- ²⁸ A. Steffens, M. Friesdorf, T. Langen, B. Rauer, T. Schweigler, R. Hübener, J. Schmiedmayer, C. A. Riofrio, and J. Eisert, *Nature Comm.* **6**, 7663 (2015).
- ²⁹ D. J. Luitz, N. Laflorencie, and F. Alet, *Phys. Rev. B* **91**, 081103 (2015).
- ³⁰ R. Singh, J. H. Bardarson, and F. Pollmann, “Signatures of the many-body localisation transition in the dynamics of entanglement and bipartite fluctuations,” ArXiv:1508.05045.
- ³¹ M. Serbyn, Z. Papić, and D. A. Abanin, *Phys. Rev. X* **5**, 041047 (2015).
- ³² M. Serbyn, M. Knap, S. Gopalakrishnan, Z. Papić, N. Y. Yao, C. R. Laumann, D. A. Abanin, M. D. Lukin, and E. A. Demler, *Phys. Rev. Lett.* **113**, 147204 (2014).
- ³³ D. Roy, R. Singh, and R. Moessner, *Phys. Rev. B* **92**, 180205 (2015).
- ³⁴ R. Vasseur, S. A. Parameswaran, and J. E. Moore, *Phys. Rev. B* **91**, 140202 (2015).
- ³⁵ S. Trotzky, Y.-A. Chen, A. Flesch, I. P. McCulloch, U. Schollwoeck, J. Eisert, and I. Bloch, *Nature Phys.* **8**, 325 (2012).
- ³⁶ M. L. Wall and L. D. Carr, “Open source TEBD,” [http://physics.mines.edu/downloads/software/tebd\(2009\)\(2013\)](http://physics.mines.edu/downloads/software/tebd(2009)(2013).).
- ³⁷ D. A. Huse, R. Nandkishore, and V. Oganesyan, *Phys. Rev. B* **90**, 174202 (2014).
- ³⁸ V. Ros, M. Mueller, and A. Scardicchio, *Nucl. Phys. B* **891**, 420 (2015).
- ³⁹ E. H. Lieb and D. W. Robinson, *Commun. Math. Phys.* **28**, 251 (1972).
- ⁴⁰ A. J. Daley, C. Kollath, U. Schollwoeck, and G. Vidal, *J. Stat. Mech.* **2004**, P04005 (2004).
- ⁴¹ W. Kirsch, “An invitation to random Schroedinger operators,” (2007), arXiv:0709.3707.
- ⁴² F. Germinet and A. Klein, *Commun. Math. Phys.* **222**, 415 (2001).
- ⁴³ M. Cramer, A. Bernard, N. Fabbri, L. Fallani, C. Fort, S. Rosi, F. Caruso, M. Inguscio, and M. Plenio, *Nature Comm.* **4**, 2161 (2013).
- ⁴⁴ E. Jones, T. Oliphant, P. Peterson, *et al.*, “SciPy: Open source scientific tools for Python,” <http://www.scipy.org/> (2001).

Appendix A: Numerical details

In this appendix, we present the details of our numerical simulations. Our results mainly rely on a matrix-product state simulation based on a TEBD code³⁶, so an instance of a tensor network state simulation. In order to corroborate the results, we have furthermore employed an exact diagonalisation code⁴⁴ that uses the particle number symmetry and keeps track of the time evolution with a Runge-Kutta integration scheme. For the non-interacting case, further checks were performed by an explicit simulation of the dynamical evolution of the covariance matrix, which takes a particularly easy form in this case.

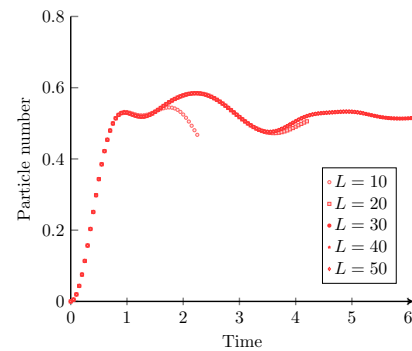


FIG. 3. Finite size scaling for the evolution of particle density in the middle of the chain for a typical disorder realisation. For $L = 10, 20$ an exact diagonalisation code was used. The other system sizes are simulated with a TEBD code³⁶.

For short times and the system sizes that can be achieved with exact diagonalisation, the codes agree up to a negligible error, thus also demonstrating that the chosen step size in the 5-th order Trotter decomposition used in TEBD³⁶ of τ_{step} does

not produce significant errors. This leaves only two potential error sources, the fact that numerics necessarily simulate a finite system and the possibility of discarded weights accumulating over time.

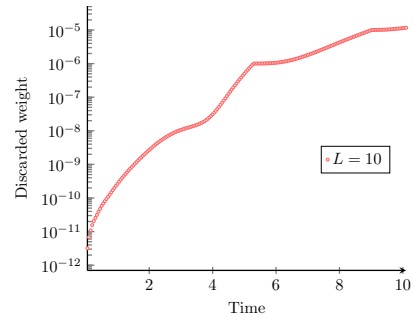


FIG. 4. Evolution of the discarded weight. This plot varies strongly depending on the chosen disorder realisation. From the 100 realisations used for the averaged plots, the realisation with the largest discarded weights is shown here.

Performing a finite size scaling, we find that comparably small systems are already indistinguishable from the thermodynamic limit for the quantities considered here, see Fig. 3. This is in agreement with the very slow growth of Lieb-Robinson cones expected in these disordered systems. To be on the safe side, we nevertheless carried out our numerical analysis on systems with $L = 80$ sites and open boundary conditions.

Having demonstrated that the considered system size is indistinguishable from the thermodynamic limit only leaves the discarded weight as potential error source, see Fig. 4. The time evolution of this quantity, which is directly connected to spatial entanglement entropies, depends strongly on the chosen disorder realisation. In order to keep this discarded weight small enough, we increase the bond dimension in the simulation in a three-step procedure up to $d_{\text{Bond}} = 350$, which is sufficient to guarantee a discarded weight smaller than $2 \cdot 10^{-5}$ for all disorder realisations.